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Insights into volcanic conduit flow from an open-source numerical model

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[1] Numerical models that calculate the fluid dynamics of explosive volcanic eruptions have been used with increasing frequency to understand volcanic processes and evaluate volcanic hazards. Yet those who develop such models rarely make them publicly available so that they can be verified, used, and possibly improved by other scientists. In this paper I present a visual, interactive, open-source numerical model that calculates steady state flow of magma and gas in vertical eruptive conduits and contains user-friendly utilities for quickly determining physical, thermodynamic, and transport properties of silicate melts, H_2O gases, and melt-gas-crystal mixtures. The model represents an advance over previously published conduit models by incorporating a non-Arrhenian viscosity relation for hydrous silicate melts, a relation between viscosity and volume fraction of gas that depends on Capillary number, and adiabatic temperature changes in the mixture using established thermodynamic relations for melts and H_2O gas, respectively. Volcanologists who have not had access to conduit models have frequently approximated conduit flow using an analytical equation for incompressible, laminar, Newtonian pipe flow, which predicts that the mass flux is proportional to the fourth power of conduit radius and inversely proportional to mixture viscosity. The model presented here, which is not much more difficult to use than a back-of-the-envelope calculation, shows that the pipe-flow approximation significantly overestimates the sensitivity of mass flux to both conduit radius and mixture viscosity. Results from the model also show that viscous heating in the lower conduit, which is not considered in most other models, may increase the mass flux of large silicic eruptions by several percent and decrease the viscosity of the mixture at the fragmentation depth by a few tens of percent.

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1. Introduction

[2] Volcanology, like most fields of Earth Science, has trended increasingly toward description of phenomena in quantitative terms that can be mathematically modeled. A reflection of this fact is the number of scientific papers using the terms "volcanic" and "model" (as judged by a Georef

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search), which amounted to less than a dozen prior to 1960; then to 83, 324, 754, and 1746 for each succeeding decade thereafter. Since 1995, a similar explosion has taken place in the number of books published on quantitative physics of volcanology [e.g., *Gilbert and Sparks*, 1998; *Sparks et al.*, 1998] or numerical modeling of volcanic processes [e.g., *Freundt and Rosi*, 1998; *Pshenichny et al.*, 2000; *Dobran*, 2001]. The trend to explain volcanic phenomena in quantifiable terms results both from an improved understanding of the processes that govern those phenomena (thanks largely to better observations and measurements) and an improved capability, with 21st century computer power, to solve numerically intensive problems that govern those processes. The results of such modeling studies, however, have generally been conveyed in the time-honored form of oral presentations or journal articles.

[3] For volcanologists who are not modelers themselves, the conventional method of conveying these advances is a source of frustration. Despite their simplifications, numerical models incorporate many complex interrelationships that cannot be easily elucidated in a paper (imagine trying to describe in writing the idiosyncrasies of a word processing program to a reader who has never used it). At best, a journal-length paper can include a few examples that illustrate the effect of parameters *A*, *B*, or *C* on a given outcome. However, for a model containing a dozen or more parameters, the full richness of relationships cannot be appreciated without simply using the model. A second shortcoming of traditional presentations is that they cannot provide enough information to assure skeptical readers or listeners that the model accurately reproduces observed phenomena (except in the specific cases illustrated), or even that it is coded correctly. These facts, along with the arcane style in which some models are explained, have contributed to the skepticism of nonmodeling volcanologists regarding the usefulness or validity of modeling results.

[4] In the digital age, these shortcomings may be alleviated if the model itself, including source code, is presented as part of the publication. The publication of open-source models places a higher standard of accountability on modeling scientists, and advances the discipline by providing numerical tools for others to build on. Curious scientists who experiment with well-documented codes can evaluate their weaknesses and improve on them.

[5] This paper describes an open-source numerical model for flow of magma and gas in eruptive conduits during steady state pyroclastic eruptions. The program was written in Fortran 90 and can be operated on any platform that has such a compiler. To make the program more user-friendly, a graphical user interface was coded in Visual Basic that runs on all Microsoft Windows®-based computers. (Use of trade names does not imply endorsement by the U.S. Geological Survey.) The interface allows users to quickly vary parameters such as conduit diameter or magma-gas content and view graphed output. Below I describe details of the program and some examples of its use.

2. Background

[6] Volcanic eruptions encompass a wide range of styles that result in large part from the type and amount of magma erupted and its flow path to the surface. The initial composition, temperature and pressure of the magma, as well as the physical and chemical changes it experiences en route to the surface, affect such eruption characteristics as ejection velocity, the degree to which eruptive plumes ascend buoyantly or collapse, and the generation of pyroclastic flows and surges. Studying volcanic conduit flow has been a key factor in understanding, and perhaps ultimately predicting, the nature of hazards associated with volcanic eruptions.

[7] Pyroclastic eruptions that persist for hours without visible changes include Plinian, sub-Plinian, and lava fountaining events. These events discharge hundreds to millions of cubic meters of magma per second ($\sim 10^5 - 10^9$ kg s^{-1}) [*Sparks et al.*, 1998] and represent the high end of the spectrum of volcanic mass fluxes. Since at least the 1950s, the conduit processes that determine the characteristics of such eruptions have been qualitatively understood (Figure 1): as magma rises from a depth of kilometers, its pressure decreases and gas bubbles exsolve. With further decompres-

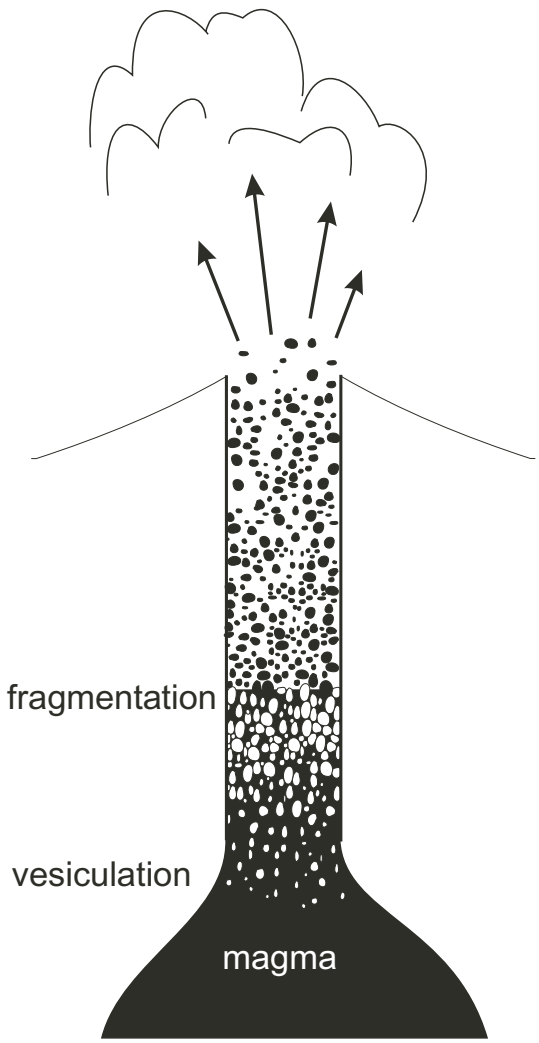


Figure 1. Schematic illustration of the processes taking place within an eruptive conduit. As the magma rises, bubbles come out of solution and expand, eventually breaking the magma apart (fragmenting it) into particles or droplets, which are entrained in an upward stream of gas.

sion the mixture accelerates and expands to a foam whose internal pressure breaks the melt into fragments. The mixture leaves the vent as a gas-particulate jet whose velocity is controlled either by the sonic velocity of the mixture (in vents that do not flare sufficiently to allow the mixture to decompress to 1 atm) or by the degree to which enthalpy can be efficiently converted to kinetic energy [*Wilson*, 1980; *Mastin*, 1995a].

[8] The first numerical model of volcanic-conduit flow was presented by *Wilson et al.* [1980], who idealized the erupting mixture as a single homogeneous fluid whose properties were bulk averages of the individual phases. Wilson et al. neglected heat and mass transfer through the conduit walls, temperature changes within the conduit, and variations in mixture viscosity associated with bubble growth or with exsolution of gases. Finally, Wilson et al. assumed that flow properties could be averaged at any given depth, reducing the problem to one of one-dimensional flow.

[9] Since 1980, dozens of papers have modeled eruptive-conduit flow. Major advances include consideration of (1) separated flow between magma and gas [*Vergniolle and Jaupart*, 1986; *Dobran*, 1992; *Papale and Dobran*, 1993], (2) variations in viscosity, density, and gas solubility with melt composition [*Papale and Dobran*, 1993], (3) gas loss to conduit walls [*Jaupart and Allègre*, 1991; *Woods and Koyaguchi*, 1995], (4) adiabatic temperature changes [*Buresti and Casarosa*, 1989; *Mastin*, 1995b], (5) kinetics of bubble growth [*Proussevitch and Sahagian*, 1998], (6) transient conduit flow [*Proussevitch and Sahagian*, 1998], (7) variations in mixture viscosity with volume-fraction gas [*Dobran*, 1992], (8) exsolution of multiple gas species [*Mastin*, 1995b; *Papale*, 1996; *Mastin*, 1997], a strain rate controlled fragmentation criterion [*Papale*, 1999], (9) incomplete fragmentation [*Papale*, 2001], (10) two-dimensional conduit flow [*Massol et al.*, 2001], and (11) effects of lava ponds around the eruptive vent [*Wilson et al.*, 1995]. Above the depth at which fragmentation occurs, additional models have considered time-dependent propagation of shock and decompression waves [*Turcotte et al.*, 1990; *Wohletz and Valentine*, 1990; *Melnik and Slezin*, 1994; *Ramos*, 1995, 1999].

[10] Despite these advances, many first-order processes have not yet been incorporated into volcanic conduit models. Our ability to realistically model conduit flow is limited by our incomplete knowledge of subsurface conduit geometry, conduit-wall strength, mechanisms of magma fragmentation, rheology of melt-bubble-crystal mixtures, and nucleation- and growth-kinetics of bubbles and crystals, among other factors. Future experiments and better theory may lead to better

characterization of mixture rheology and phase changes within conduits. However, conduit geometry may never be known well enough to constrain flow properties except within broad limits dictated by the strength and ambient stress of the host rock.

[11] Given these limitations, conduit models in their current state are useful not so much as predictors of eruption characteristics, but as tools that illustrate how such characteristics might change as parameters such as temperature or melt composition vary. To the extent that conduit-flow models have been used to simulate real, historical eruptions [e.g., *Dobran*, 1992; *Papale and Dobran*, 1993], they have been used to explain changes in eruptive characteristics associated with variations in a few parameters (primarily magma composition) while holding others constant.

3. Model Description

[12] The program, called "Conflow," is described in detail by *Mastin and Ghiorso* [2000] (available at http://vulcan.wr.usgs.gov/Projects/Mastin). Here I present an abbreviated description.

[13] Conflow models flow in eruptive conduits by assuming that (1) the flow is steady, (2) the conduit is vertical and circular in cross section, (3) temperature, pressure, and velocity are the same for all phases at a given depth, (4) flux of heat and gas through the conduit wall are negligible relative to their flux up the conduit, (5) the flow is one-dimensional (i.e., flow properties at any depth can be averaged across the conduit's cross section), (6) no work is done between the flowing magma and the surroundings, and (7) prior to fragmentation, gas exsolves fast enough to maintain chemical equilibrium with the melt. After fragmentation, no further gas exsolution is calculated under the assumption that the decompression rate greatly exceeds that at which gas can exsolve.

[14] Among the most limiting of these assumptions is 3, which implies that the melt can be modeled as a homogenous fluid. Although this assumption was made in classic flow models [e.g. *Wilson et al.*, 1980], some more recent models [e.g., *Dobran*, 1992; *Papale and Dobran*, 1993; *Papale et al.*, 1998] solve separate equations for the mass and momentum of the gas and melt+crystal phases. Separated-flow models are more accurate than Conflow above the depth of fragmentation, and in certain cases where large bubbles rise through slowly ascending basalt [*Vergniolle and Jaupart*, 1986; *Mastin and Ghiorso*, 2000]. Using a separated flow model, *Dobran* [1992] calculated exit velocities that were 15–25% higher, and exit pressures 50–75% lower, than calculated by a homogeneous model for the same conditions. The inaccuracy of the homogenous assumption does not significantly detract from the usability of this model in evaluating the effect of flow parameters on eruption dynamics. Moreover, the errors introduced by this assumption are probably smaller than those caused by oversimplifications in melt rheology, fragmentation criterion, and the assumption of one-dimensional flow. Of perhaps greater significance is this model's assumption of one-dimensionality, which glosses over critical effects that no doubt take place at the edges and centers of conduits. To date, all conduit-flow models that consider properties both below and above the fragmentation depth are one-dimensional. I discuss some limitations of this assumption later.

3.1. Governing Equations

[15] Given the above assumptions, the equations for mass and momentum conservation are as follows:

$$\frac{d(\rho u A)}{dz} = 0 \tag{1}$$

$$\rho u \frac{du}{dz} = -\rho g - \rho u^2 \frac{f}{r} - \frac{dp}{dz}. \tag{2}$$

The terms ρ and u are the bulk density and velocity of the erupting mixture, respectively; A is the cross-sectional area of the conduit; z is vertical position (positive being upward); g is gravitational acceleration; p is pressure; f is a dimensionless factor whose value controls frictional pressure loss in the conduit [*Bird et al.*, 1960]; and r is conduit radius. Through a series of substitutions and rearrange-

Table 1. Listing of the Dependent and Independent Variables, the Boundary Conditions, Input Parameters, and the Parameters on the Right-Hand Side of Equations (3) and (5), Specifying the Terms on Which Those Parameters Depend

Variable Type	Variable(s)	Explanation
Independent variable	z	elevation in conduit
Dependent variables	$p(z)$	pressure (for solutions using equation (3))
	$A(z)$	conduit cross-sectional area (for solutions using equation (5))
Boundary conditions	p	pressure at base of conduit
	p or M	pressure or Mach number at top of conduit
Input variables	p_o, p_f	pressure (MPa) at base, top of conduit[a]
	T_o	temperature (C) at base of conduit
	u_o	velocity (m/s) at the base of the conduit[b]
	w	H_2O content in mixture (wt%)
	z_o, z_f	depth (m) at base, top of conduit
	D_o, D_f	conduit diameter (m) at base, top[c]
	g	gravitational acceleration (m/s^2)
	X	Chemical composition of melt, specified as wt% SiO_2, Al_2O_3, Fe_2O_3, FeO, MgO, CaO, TiO_2, Na_2O, K_2O
	$\hat{v}_x$	volume percent crystals in melt[d]
	crystal type	used to calculate crystal density, specific heat.

[a] When solving equation (5), Conflow assumes that pressure varies linearly with depth between these values.
[b] When solving equation (3), the specified value of u_o is the starting value in an iteration sequence that adjusts u_o until one of the two upper boundary conditions is met. When solving (5), u_o is not adjusted.
[c] When solving equation (3), Conflow assumes that conduit diameter varies linearly with depth between these values. When solving (5), Conflow ignores the value of D_f.
[d] This term differs from v_x, the volume fraction of crystals in the mixture.

ments (described by *Mastin and Ghiorso* [2000]), equations (1) and (2) can be combined to give

$$-\frac{dp}{dz} = \frac{\rho g + \rho u^2 \frac{f}{r} - \frac{\rho u^2}{A}\frac{dA}{dz}}{1 - M^2}, \qquad (3)$$

where M is the Mach number of the flowing mixture, i.e., its speed normalized to the speed of sound (c) in that mixture ($M \equiv u/c$).

[16] The first and second terms in the numerator on the right-hand side (RHS) of equation (3) reflect pressure losses associated with weight of the magma and friction, respectively; the third term reflects pressure loss associated with changes in conduit cross-sectional area. The denominator reflects the effect of sonic velocity, and this effect is dramatic. As the accelerating mixture approaches the sonic velocity (several tens to a few hundred meters per second in volcanic gas-tephra mixtures), the denominator on the RHS of equation (3) approaches zero. The mixture can accelerate from subsonic to supersonic velocity only at a point in the conduit where all terms in the numerator cancel one another, i.e., at a constriction in the conduit, or at the beginning of a flaring section. In a conduit of constant cross-sectional area, the mixture can never accelerate to a speed greater than the sonic velocity; the mixture will exit the vent at $M = \sim 1$, $p > 1$ atm and will rapidly decompress above the ground surface through a series of shock waves. Flows through nozzles and conduits in which the flow rate is limited by sonic velocity are termed "choked" [e.g., *Liepmann and Roshko*, 1957, p. 53]; the effects of choked flow on eruptive mass flux are detailed later.

[17] In Conflow, the magma pressure, temperature, velocity, and composition (including abundance and type of crystal phases) are specified at the base of the conduit as well as the conduit length, diameter, and variation in diameter with depth (Table 1). From this information, the value of each term on the (RHS) of equation (3) can be determined at the base of the conduit, dp/dz can be evaluated, and the pressure can be extrapolated to a higher elevation. At the new elevation, once the pressure is known, all of the terms on the RHS of equation (3) can again be calculated and so on. Conflow integrates equation (3) numerically using a Cash-Karp method with automatic quality control that adjusts the vertical step size to concentrate

calculations at points where properties are changing most rapidly [*Press et al.*, 1992].

[18] Using values of u and z at each depth, the enthalpy of the mixture (h) at that depth is obtained using the equation for energy conservation [*Moran and Shapiro*, 1992, p. 126]:

$$h = h_0 + \frac{1}{2}\left(u_0^2 - u^2\right) + g(z_0 - z), \tag{4}$$

where h_0, u_0, and z_0 are the mixture enthalpy, velocity, and elevation at the base of the conduit. Conflow calculates h_o from temperature-enthalpy relations derived by *Ghiorso and Sack* [1995] for the melt and *Haar et al.* [1984] for the gas (assumed to be pure H_2O; see *Mastin and Ghiorso* [2000, 2001] for details). Once h is determined, Conflow uses the same *T-h* relations to obtain a temperature at this depth. The fact that the new enthalpy is calculated after each integration step implicitly assumes that changes in enthalpy do not significantly affect the flow properties. The Cash-Karp integration routine checks this assumption by simultaneously integrating over two different dz steps and then comparing the results. If the results differ significantly, it reduces the dz step and reintegrates.

[19] Upon reaching the Earth's surface, the flow properties must satisfy one of two boundary conditions: (1) the exit pressure must equal 1 atmosphere (if pressure at the base of the conduit is not sufficient to accelerate the mixture to sonic velocity); or (2) the Mach number must equal 1 (i.e., flow is "choked"). If neither condition is met, Conflow adjusts the velocity at the base of the conduit and re-integrates until one of these conditions is satisfied.

3.2. Conduit Geometry

[20] The above discussion brings out a critical weakness of this and other conduit models: in order to integrate equation (3), the geometry of the conduit must be known. Yet our knowledge of conduit geometry is crude at best and has been inferred only by inspection of ancient, eroded volcanoes, or through indirect (and approximate) geophysical methods. The vast majority of conduit modelers [e.g., *Dobran*, 1992; *Papale and Dobran*, 1993; *Sparks et al.*, 1994; *Papale et al.*, 1998] simply assume a cylindrical conduit and study effects of melt composition, gas content, temperature, etc. on conduit pressure or fragmentation depth using this assumption.

[21] The conduit pressure (which affects fragmentation depth) is constrained to some degree by the strength of the host rock and the ambient stress field. Some, perhaps even most, silicic eruptive conduits may be elongate or dike-shaped [e.g., *Eichelberger and Carrigan*, 2000] and therefore have an internal pressure influenced by the least-compressive horizontal stress (dike-like geometries have long been inferred for many mafic vents). However, even within cylindrical conduits, high pressure during an eruption will likely be relieved by hydrofracture of conduit walls, and low pressure will be inhibited by conduit-wall collapse or constriction.

[22] *Wilson et al.* [1980] and *Giberti and Wilson* [1990] considered the variation in flow properties within a conduit whose pressure profile was fixed (i.e., lithostatic, or some similar value). Conflow considers this possibility by rearranging equation (3) to solve for changes in conduit cross-sectional area, given a specified pressure gradient:

$$\frac{dA}{dz} = \frac{A}{\rho u^2}\left[\frac{dp}{dz}\left(1 - M^2\right) + \rho g + \frac{f\rho u^2}{r}\right]. \tag{5}$$

As with equation (3), all terms on the right-hand side of the equation are known or can be calculated from input conditions at the base of the conduit. For a given pressure gradient, the conduit geometry can be solved by a straight one-dimensional integration. The specified boundary condition at the surface (1 atm) is automatically satisfied in the first integration run, eliminating the need for multiple iterations and adjustments to the input velocity. Solutions to equation (5) generally produce a flaring vent near the surface with supersonic exit velocities.

3.3. Material Properties

[23] The density, velocity, Mach number, and friction factor (f) must all be known at a given depth to integrate equations (3) and (5). The constitutive

equations that govern these relationships are given by *Mastin and Ghiorso* [2000]. In qualitative terms, they are as follows.

3.3.1. Density

[24] At any depth, the density of the mixture is a weighted sum of the densities of the gas, melt, and crystals, times their respective volume fractions in the mixture. The densities of the melt and gas are calculated using methods of *Ghiorso and Sack* [1995] and *Haar et al.* [1984]; the density of crystals is assumed constant. The volume fraction of each phase is calculated from their mass fractions and their densities. The mass fraction of crystals in the mixture is held constant, while the mass fractions of exsolved gas and melt are recalculated at each depth as water exsolves.

3.3.2. Velocity

[25] The mixture's velocity is determined from equation (1), which states simply that in a steady state eruption, the product of density, velocity, and conduit cross-sectional area is the same all depths. If the first and third properties are known, the second can be calculated.

3.3.3. Mach number

[26] The Mach number is the velocity of the mixture divided by its sonic velocity. The sonic velocity is given by $(\partial p/\partial \rho)_S$, where the subscript S denotes constant-entropy conditions. This partial derivative is known for each of the melt, gas, and crystals from relations by *Ghiorso and Sack* [1995], *Haar et al.* [1984], and *Berman* [1988], respectively. The partial derivative for the mixture is calculated by summing the partial derivatives for each separate phase, multiplied by their respective volume fractions in the mixture.

3.4. Constitutive Relations

3.4.1. Friction factor

[27] The friction factor (f) is a dimensionless parameter that relates the frictional momentum loss within the conduit to the momentum flux through the conduit. Under laminar flow conditions, which characterize nearly the entire conduit below the depth of fragmentation, $f = 16/Re = 16\eta/\rho uD$, where Re is the Reynolds number for flow, η is the viscosity of the erupting mixture, and D is conduit diameter. Under turbulent-flow conditions, which exist primarily above the fragmentation depth, the friction factor is controlled by conduit-wall roughness and can be regarded as an empirical constant with a value between ~0.001 and 0.02 [*Bird et al.*, 1960, p. 186]. Variations in the value of this constant do not significantly affect flow profiles within volcanic conduits [*Mastin*, 1995b].

[28] Because the viscosity of silicate melts ranges over several orders of magnitude [*Shaw*, 1972], it is the most significant parameter in determining the dynamics of conduit flow. For melts containing less than 70% SiO_2 among the anhydrous components, Conflow calculates viscosity using relations of *Shaw* [1972], which uses a relatively simple Arrhenian relationship in which the log of viscosity is inversely proportional to absolute temperature. For more siliceous melts, Conflow uses the relation of *Hess and Dingwell* [1996], which uses more realistic, non-Arrhenian temperature dependence and more accurately incorporates the effect of dissolved water.

[29] To account for effects of crystallinity, Conflow uses the *Marsh* [1981] calibration of the Roscoe-Einstein equation, in which viscosity increases exponentially toward infinity as the crystal-volume fraction in the melt approaches 60%. At crystal volume fractions exceeding ~5–30% by volume (depending on crystal shape), these suspensions develop a yield strength [e.g., *Larson*, 1999, p. 267] which is not considered by Conflow or any other steady state conduit model. Some classic Plinian eruptions (e.g., Mount St. Helens, 1980 and Pinatubo, 1991, described later) have crystallinities that greatly exceed this threshold.

[30] Conflow considers the effect of bubbles on mixture viscosity (η) using the relation $\eta = \eta_{m+x}(1 - v_g)^n$, where η_{m+x} is the viscosity of the liquid-crystal suspension, v_g is the volume fraction gas in the mixture, and n is a parameter whose value ranges from -1 to 1 depending on the Capillary number (Ca): for $\log(Ca) \rightarrow -\infty$, $n \rightarrow -1$ whereas for $\log(Ca) \rightarrow \infty$, $n \rightarrow 1$. The Capillary number (Ca) is defined as $Ca \equiv \varepsilon\eta_{m+x}d/\gamma$, where ε is the

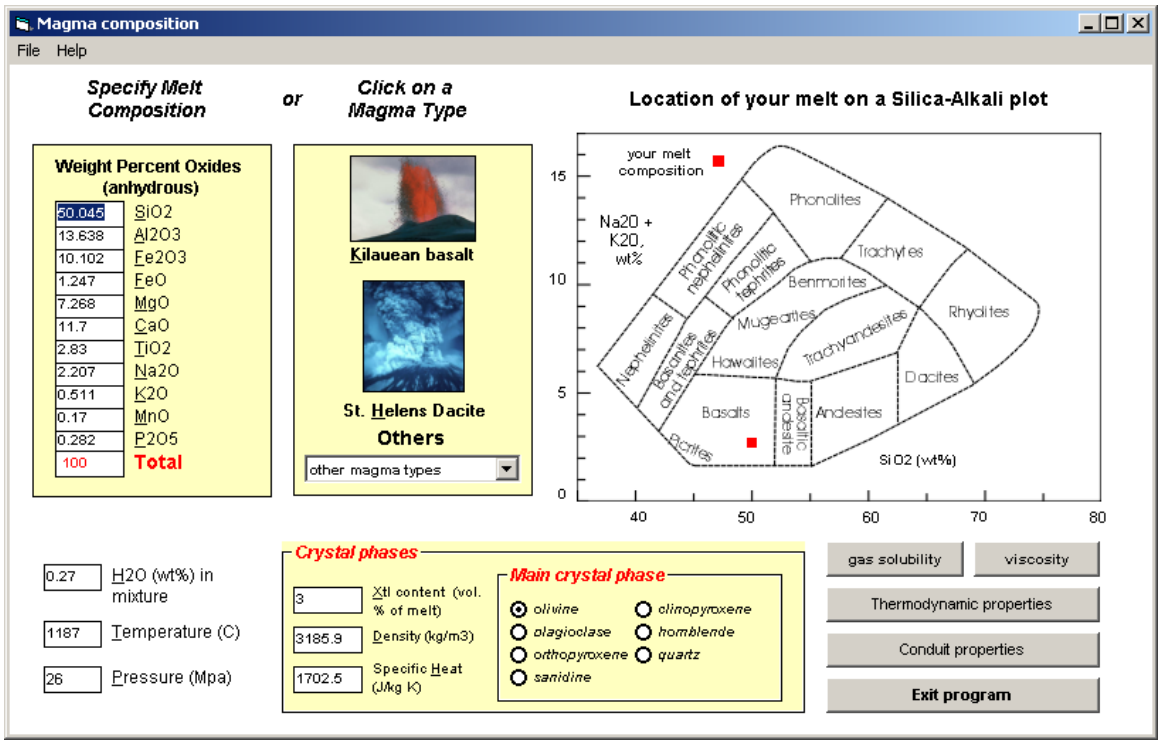


Figure 2. Magma composition page of Conflow. In this page, users may specify the composition, pressure, and temperature of a melt and the type and percentage of the dominant crystal phase.

shear-strain rate of the bubbly mixture, η_{m+x} is the viscosity of the liquid-crystal mixture, d is average bubble diameter, and γ is interfacial-surface tension between the melt+crystals and the gas. Low-viscosity mafic magmas tend to have low Capillary numbers and therefore have mixture viscosities that increase with volume-fraction gas [*Manga et al.*, 1998]. Silicic magmas have higher Capillary numbers and their viscosity may decrease with increasing gas content. Suspensions that contain more than a few tens of volume-percent bubbles develop viscoelastic, shear-thinning, and other non-Newtonian characteristics [*Larson*, 1999, p. 413; *Rust and Manga*, 2000] that are not accounted for by this or any other volcanic conduit model currently in use.

[31] Conflow estimates the average Capillary number at a given depth and uses it to adjust viscosity for volume fraction gas (details are provided by *Mastin and Ghiorso* [2000]). When using a strain rate fragmentation criterion (described in section 3.4.2), variations in mixture viscosity due to Capillary number can significantly alter the depth of the fragmentation front and the threshold vesicularity at which fragmentation occurs [*Mastin and Ghiorso*, 2000].

3.4.2. Fragmentation criterion

[32] The threshold at which the bubbly melt breaks into particles coincides with a drop of several orders of magnitude in mixture viscosity and corresponding changes in velocity and pressure. Conflow uses a classic criterion of 75% vesicularity for this transition [*Sparks*, 1978], acknowledging that this criterion is an oversimplification. With a minor modification to its source code, Conflow may employ the more physically based criterion set forth by *Papale* [1999], which is based on relationships between strain rate and viscous relaxation time, and effects of non-Newtonian shear thinning established by *Dingwell and Webb* [1989, 1990] and *Webb and Dingwell* [1990]. The standard version of Conflow does not use this strain rate criterion because it is highly sensitive to certain parameters such as elastic modulus and mixture viscosity, whose exact values are the subject of some uncertainty [*Mastin and Ghiorso*, 2000]. The

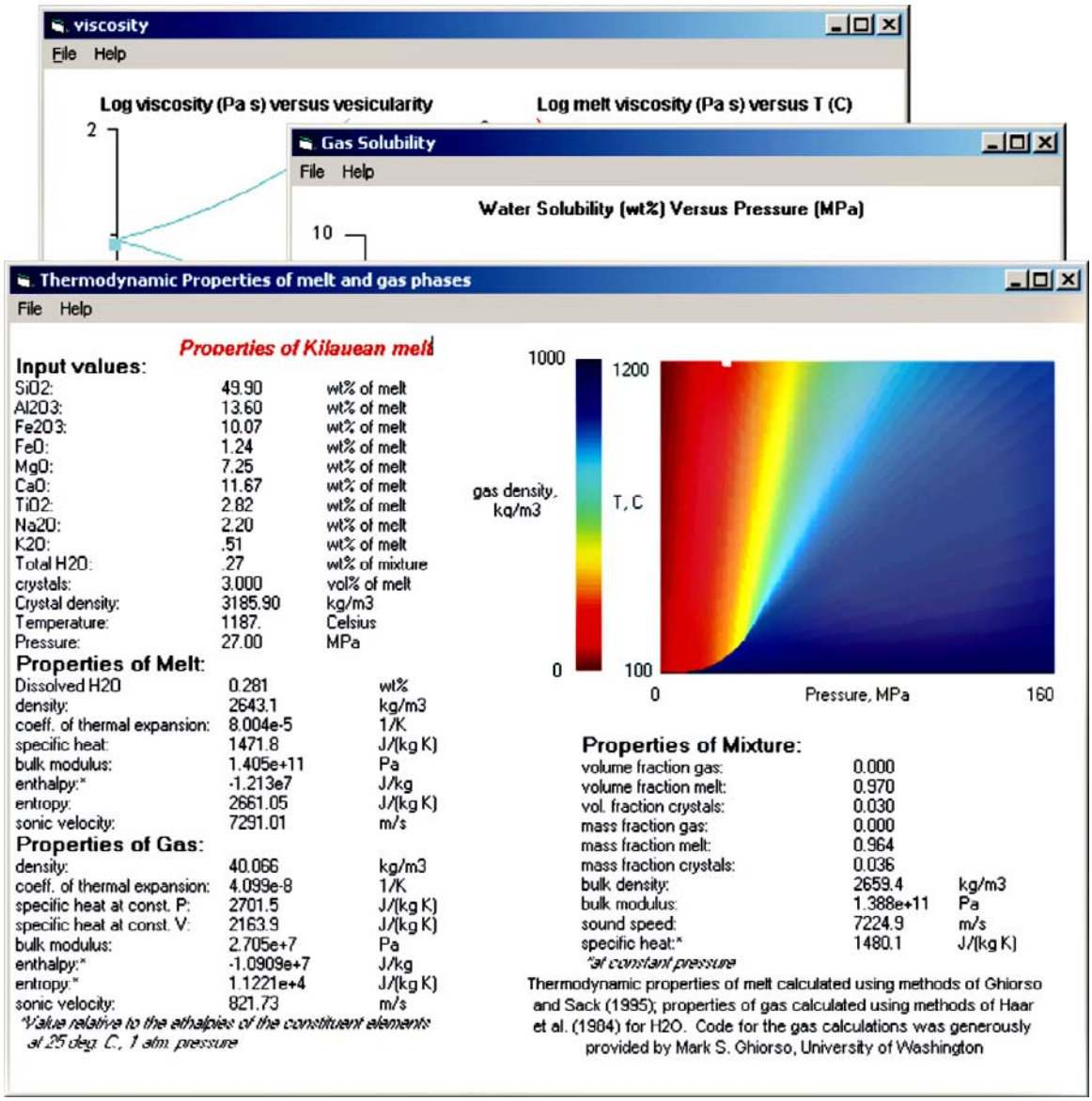


Figure 3. Conflow pages that give the gas solubility, viscosity, and thermodynamic and physical properties of the magma.

mechanism of magma fragmentation is the subject of active research [e.g., *Alidibirov and Dingwell*, 1996, 2000; *Zimanowski et al.*, 1997; *Dingwell*, 1998; *Ishihara et al.*, 2002]. The next few years will likely see significant improvements in the capability of conduit models to realistically incorporate fragmentation.

4. Using Conflow

[33] When launched, Conflow opens a window (Figure 2) in which users specify the pressure, temperature, and composition of the mixture at the base of the conduit. Melt composition is set by either entering the weight percent of constituent oxides (in the left column) or choosing from several preentered magma types (second column from left). The percentage and type of the main crystal phase can be specified in the lower box; when chosen, the specific heat and density of that mineral at the given pressure and temperature is displayed using relations from *Berman* [1988]. Thermodynamic properties for the melt, gas, and melt-gas-crystal mixture can be viewed (Figure 3)

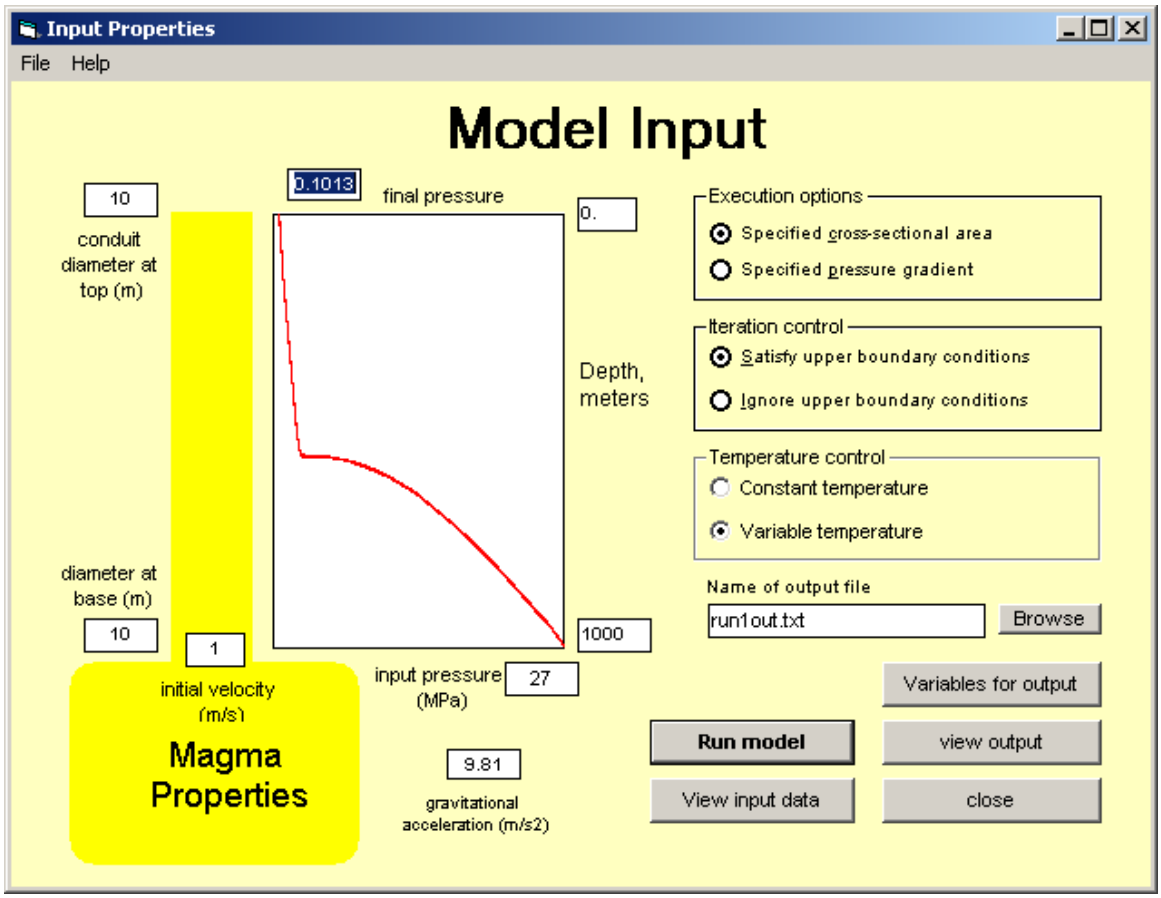


Figure 4. Conflow page that allows users to specify the geometry and boundary conditions of the conduit.

through a command button on the opening page, as can the water solubility of the melt and the viscosity of the melt, melt+crystal, and melt+gas+crystal mixtures. Conduit properties are specified in a separate window (Figure 4).

[34] Upon pressing the "run" command button in the conduit properties window, Conflow writes the input parameters to an ASCII file, opens a DOS window, and runs the conduit model itself. The conduit-flow calculations are performed through an executable program (called "conex") compiled from Fortran 90 source code (available at http://vulcan.wr.usgs.gov/Projects/Mastin/). Conex will run on any operating system for which a Fortran 90 compiler exists; for non-Microsoft operating systems, input parameters are changed by editing the ASCII input file rather than through a visual interface.

[35] Results are written to an ASCII file and can be viewed in tabulated form or plotted and compared with results from preceding runs (Figure 5). Typical model runs are completed in 10–30 s, allowing users to quickly evaluate the effect of various input conditions on eruption dynamics.

5. Sample Applications

[36] Below are some applications of the program to issues regarding eruption dynamics that have been posed in the literature.

5.1. Effects of Temperature and Volatile Content on Flow Dynamics

[37] For more than a century it has been known that volatile content and melt composition strongly affect the dynamics of volcanic eruptions: silicic magmas are high in viscosity and gas content, and tend to produce explosive eruptions with high magma-flux rates and high exit velocities. Mafic eruptions tend to have lower volatile

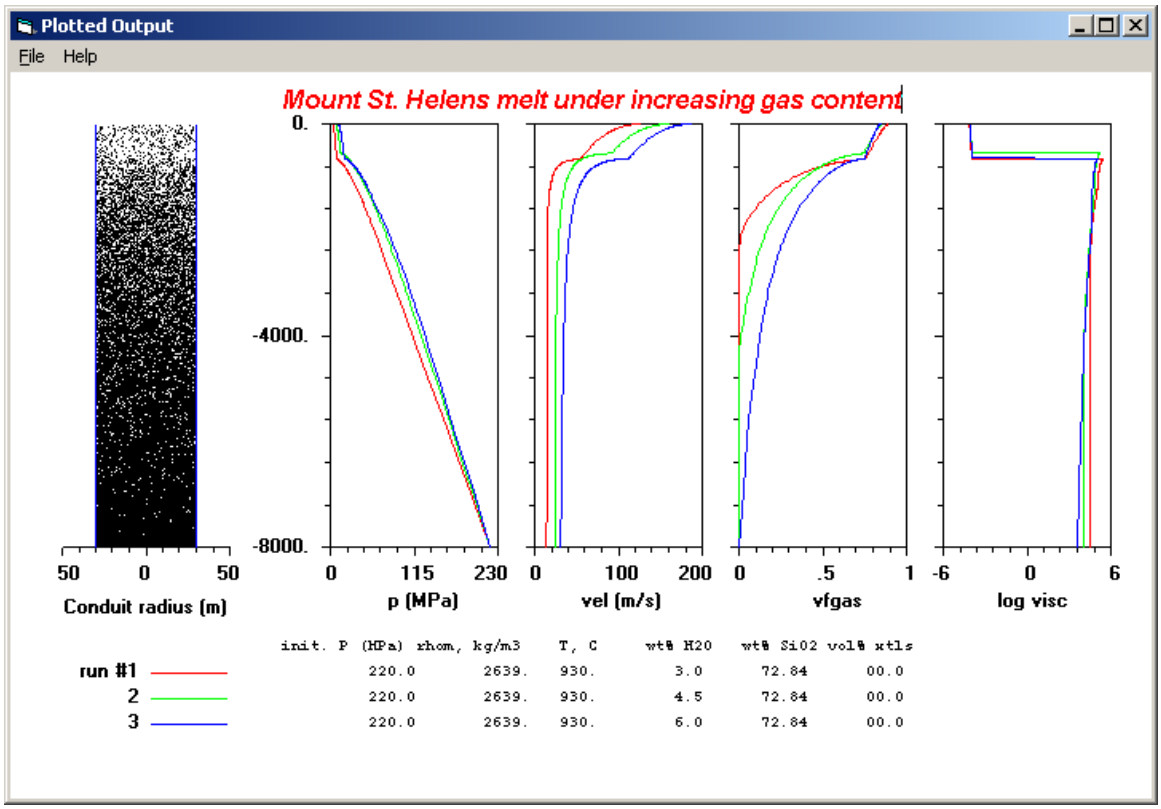


Figure 5. Example of plotted output using Mt. St. Helens melt composition (given in Table 2) with no crystals and initial pressure and temperature of 220 MPa and 930°C, respectively. Runs assume constant conduit diameter of 60 m and a conduit length of 8 km. For the three runs, the H_2O content in the mixture was varied from 3.0 (run 1) to 4.5 (run 2) to 6.0 wt % (run 3). Plotted parameters are (from left to right) pressure (MPa), velocity (m/s), volume fraction gas, and the log of viscosity (Pa s). On the left side of the figure is a schematic illustration of the volume fraction gas (white pixels) versus depth in the conduit for run 3.

contents, lower viscosities, and vent from the earth in lava fountains or flows having relatively low mass flux rates and exit velocities. Somewhat less appreciated are the effects of individual changes in these three parameters (for example, changes in gas content while holding melt composition constant).

[38] Using melt compositions (Table 2) from Kilauea and Mount St. Helens as examples, Table 3

Table 2. Melt Compositions Used for Sample Runs[a]

Property	Kilauea Basalt	Mt. St. Helens	Pinatubo
Temperature (C)	1150	930.	780.
SiO_2	51.96	72.84	77.53
Al_2O_3	14.21	14.57	12.81
Fe_2O_3	0.00	0.00	0.00
FeO	10.96	2.06	0.80
MgO	6.59	0.5	0.23
CaO	10.86	2.35	1.30
TiO_2	2.53	0.36	0.14
Na_2O	2.48	5.15	4.16
K_2O	0.41	2.17	2.98

[a] Basalt is a standard Kilauean composition (U.S. Geological Survey, unpublished data, 2002). Mount St. Helens melt composition taken from *Rutherford et al.* [1985] (sample SH = 084). Pinatubo composition taken from *Luhr and Melson* [1996] for white pumice. Weight percent oxides listed above neglect water.

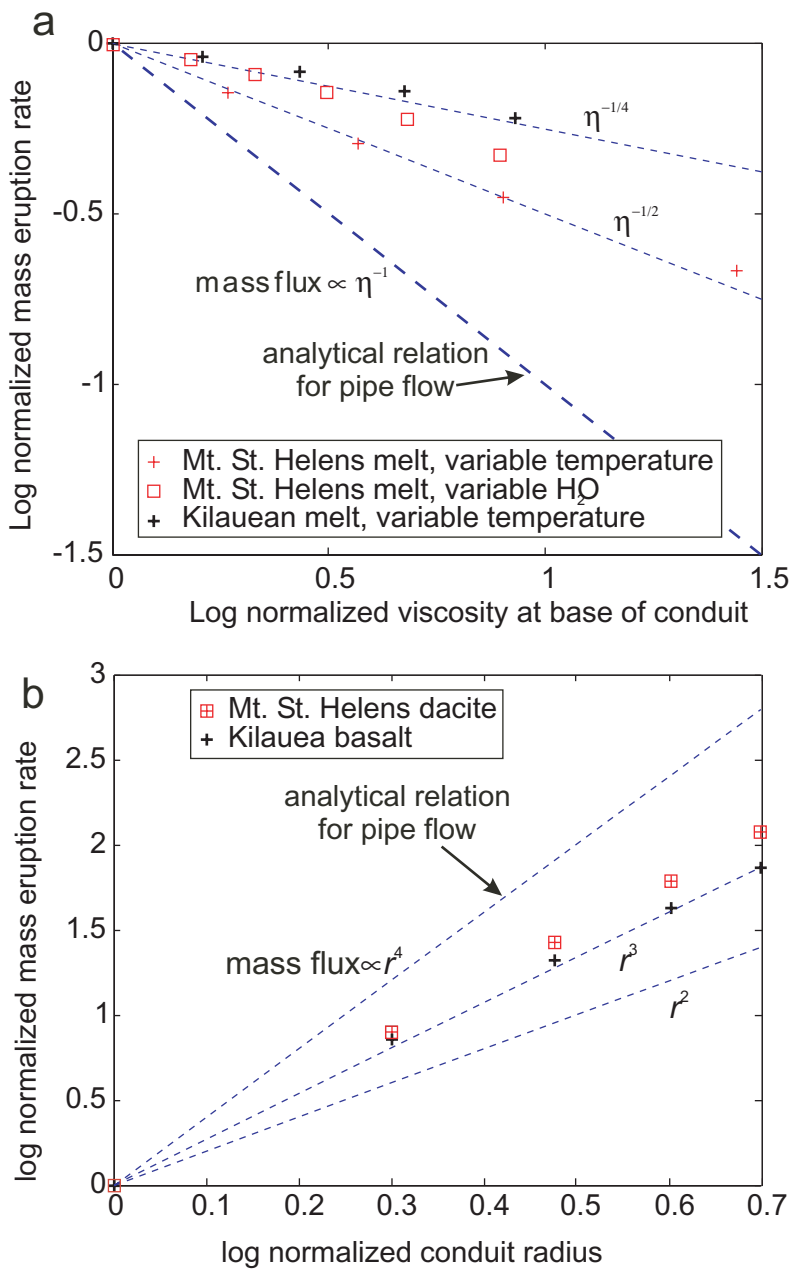
Table 3. Flow Properties Calculated by Conflow for Eruptions Involving Mount St. Helens Melt ("MSH") and Kilauean Basalt, in Which Weight Percent H_2O in the Mixture and Temperature (T_i) Were Varied[a]

Melt Type	H_2O, wt%	u_{final}, m/s	Mass Flux, kg/s	z_{frag}, m	v_g,final	T_i, celsius
MSH	2.5	127.94	6.78E+07	841	0.922	930
MSH	3	142.26	9.51E+07	751	0.902	930
MSH	3.5	155.83	1.21E+08	687	0.886	930
MSH	4	168.51	1.43E+08	654	0.875	930
MSH	4.5	180.26	1.62E+08	648	0.869	930
MSH	5	191.19	1.78E+08	672	0.864	930
MSH	5.5	201.42	1.91E+08	712	0.862	930
MSH	5.811[b]	224.37	3.10E+08	117	0.797	1000
MSH	5.922[b]	213.82	2.23E+08	528	0.845	950
MSH	5.96[b]	210.18	2.00E+08	755	0.862	930
MSH	6.036[b]	194.91	1.66E+08	1150	0.877	900
MSH	6.153[b]	199.5	1.19E+08	1918	0.915	850
MSH	6.263[b]	194.28	8.31E+07	2780	0.94	800
MSH	6.32[b]	188.56	5.59E+07	3725	0.958	750
Kilauea	0.1	18.26	2.50E+05	0	0.584	1186
Kilauea	0.25	36.78	2.64E+05	1.5	0.785	1186
Kilauea	0.5	58.07	3.77E+05	8.4	0.806	1186
Kilauea	0.664	67.14	3.67E+05	31	0.838	1100
Kilauea	0.664	67.72	3.86E+05	25.2	0.831	1120
Kilauea	0.664	68.25	4.04E+05	21.3	0.823	1140
Kilauea	0.664	68.76	4.23E+05	16.7	0.816	1160
Kilauea	0.664	69.27	4.41E+05	13.4	0.81	1180
Kilauea	0.664	69.45	4.46E+05	12	0.808	1186
Kilauea	0.664	69.78	4.57E+05	10.6	0.804	1200
Kilauea	0.75	74.9	4.81E+05	14.3	0.808	1186
Kilauea	1	89.27	5.71E+05	19.2	0.808	1186
Kilauea	1.25	101.97	6.47E+05	25.5	0.81	1186
Kilauea	1.31[b]	104.84	6.63E+05	27	0.811	1186

[a] The term u_{final} is the exit velocity, z_{frag} is the fragmentation depth, and $v_{g,final}$ the volume fraction gas at the exit.
[b] Saturation at the initial pressure.

illustrates the effects of gas content and temperature on selected conduit-flow properties. For this exercise, the crystal content was set to zero and the conduit diameters (60 m for Mount St. Helens and 4 m for Kilauean melt) were adjusted to give mass flow rates of the same order of magnitude as the Plinian eruption of Mount St. Helens on 18 May 1980 (10^7 kg s^{-1} [*Carey and Sigurdsson*, 1985]) and lava-fountain eruptions at Pu'u O'o, Kilauea during the 1980s (10^5 kg s^{-1}, [*Wolfe*, 1988]). Conduit lengths (8 and 1 km for Mount St. Helens and Pu'u O'o, respectively) were set to approximate (within ~1 km) the values estimated for these eruptions [*Pallister et al.*, 1992; *Wolfe*, 1988]. Model runs assume a cylindrical conduit and solve for pressure and other properties.

[39] The Mount St. Helens runs indicate that for flow in a cylindrical conduit, a reduction in gas content from 5.96 wt % (saturation) to 3 wt % decreases mass flow rate by ~50%, decreases final exit velocity from 210 to 142 m s^{-1}; and increases the volume fraction gas from 0.86 to 0.90 at the surface (Figure 5). The effects of varying gas content on these parameters effects were pointed out by *Papale et al.* [1998]. The overall pressure in the conduit also increases with increasing gas content, though only slightly for gas contents greater than ~4.5 wt % (Figure 5). Perhaps most surprisingly, the depth of fragmentation does not change monotonically with gas content; it decreases as gas content is increased from 2.5 to ~4.5 wt %, then increases again with further increases in gas content. This nonintuitive result is an outcome of the competing effects of gas exsolution and melt viscosity on pressure gradient, and of the variable pressures at which these mixtures reach v_g = 0.75.


a
Log normalized mass eruption rate
Log normalized viscosity at base of conduit
0
-0.5
-1
-1.5
0
0.5
1
1.5
η^−1/4
η^−1/2
mass flux ∝ η^−1
analytical relation
for pipe flow
Mt. St. Helens melt, variable temperature
Mt. St. Helens melt, variable H2O
Kilauean melt, variable temperature
b
log normalized mass eruption rate
log normalized conduit radius
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Mt. St. Helens dacite
Kilauea basalt
analytical relation
for pipe flow
mass flux∝r^4
r^3
r^2

[40] For Kilauean basalt, the relations between gas content and mass eruption rate, final exit velocity, and final volume fraction gas are qualitatively the same as for a silicic melt; however, ejection velocity (18–104 m s^{-1}) and volume fraction gas (0.58–0.81) at the exit are dramatically lower, and fragmentation depth (<25 m) dramatically shallower than for Mount St. Helens melt. All exit velocities listed in Table 1 equal the sonic velocity, even the 18 m s^{-1} value for the Kilauean melt containing only 0.1 wt % H_2O. If we assume that the kinetic energy at the exit were efficiently converted into elevation potential energy (i.e., $(1/2)u^2 = gH$, where H is height), lava-fountain heights would range from 16 to 550 m. The latter height is roughly equal to the greatest lava-fountain height witnessed during historical time at Kilauea (~530 m [*Richter et al.*, 1970]).

5.2. Laminar Pipe Flow Approximation

[41] Although steady state volcanic conduit-flow models have existed since the early 1980s, the absence of readily available programs has prompted some investigators to derive relations from simplified, back-of-the envelope equations. One example has been to employ the relation for steady state laminar flow of an incompressible fluid through a pipe, following the formula [e.g., *Gardner et al.*, 1995; *Rutherford and Gardner*, 2000]:

$$\dot{m} = \frac{g\rho_m \Delta\rho \pi r^4}{8\eta} \tag{6}$$

where $\dot{m}$ is mass flux, g is gravitational acceleration, ρ_m is the density of the erupting mixture, $\Delta\rho$ is the density contrast between the host rock and the erupting mixture, r is conduit radius, and η is mixture viscosity.

[42] The above equation implies that mass eruption rate increases with the fourth power of conduit radius and decreases with the first power of mixture viscosity. Although both of these relations are qualitatively correct, they overstate the sensitivity of mass eruption rate to these parameters under choked flow conditions. During high-flux rate eruptions, the ascent rate in volcanic conduits is limited by the sonic velocity of the mixture at the point of maximum constriction rather than by fluid viscosity and the diameter of the conduit. A few runs by Conflow (Figure 6a) illustrate that, for cylindrical conduits, mass eruption rate scales with $\eta^{-1/4}$ to $\eta^{-1/2}$, rather than η^{-1} predicted by equation (6). Similarly, mass eruption rate scales more closely with r^3 than the r^4 predicted by equation (6) (Figure 6b). These relations theoretically approach the pipe-flow predic-

Figure 6. (opposite) (a) Log of normalized mass eruption rate (kg s^{-1}) versus log normalized viscosity at the base of the conduit, for Mt. St. Helens and Kilauean melts. Thin crosses represent calculations in which viscosity of a Mt. St. Helens melt (Table 2) was varied by specifying input temperatures of (from left to right on plot) 950°, 900°, 850°, 800°, and 750°C. The viscosity and mass eruption rate were normalized to values produced under an initial magma temperature of 950 C. Squares represent Mt. St. Helens melt in which viscosity was varied by specifying initial dissolved water concentration of (from left to right) 5.658 wt % (saturated), 5, 4.5, 4, 3.5, and 3 wt %. The mass eruption rate and viscosity for these points were normalized to values calculated for an initial water content of 5.658 wt %. Thick crosses represent melt of Kilauean composition in which viscosity was adjusted by varying the initial temperature from (from left to right) 1300°, 1250°, 1200°, 1150°, and 1100°C. The heavy dashed line represents the relation between mass flow rate ($\dot{m}$) and viscosity (η) that one would expect for an incompressible fluid under laminar flow in a cylindrical pipe. The Mt. St. Helens runs assume volume percent crystals = 0, conduit diameter = 60 m, conduit length = 8 km, and basal conduit pressure = 200 MPa. Kilauean runs assume volume fraction crystals = 0, conduit diameter = 10 m, conduit length = 1 km, and pressure at the base of the conduit = 27 MPa. (b) Log of normalized mass eruption rate versus log of normalized conduit radius for runs of Kilauean basalt and Mount St. Helens dacite. Squares represent results for Mt. St. Helens dacite for (from left to right) conduit radii of 10, 20, 30, 40, and 50 m, normalized to those for a conduit radius of 10 m. Initial temperature = 930 C, volume fraction crystals in melt = 0.27, initial dissolved water = 4.6 wt % conduit length = 8 km, basal pressure = 200 MPa. Thick crosses give results for a Kilauean melt for (left to right) 1, 2, 3, 4, and 5 m conduit radii, normalized to results for a conduit radius of 1 m. Initial temperature = 1150°C, volume fraction crystals in melt = 0.03, initial dissolved water = 0.27 wt %, conduit length = 1 km, basal pressure = 27 MPa. Heavy dashed line depicts the relationship ($\dot{m} \propto r^4$) for laminar flow of an incompressible fluid through a cylindrical conduit. Lighter dashed lines depict the relationships $\dot{m} \propto r^3$ and $\dot{m} \propto r^2$.

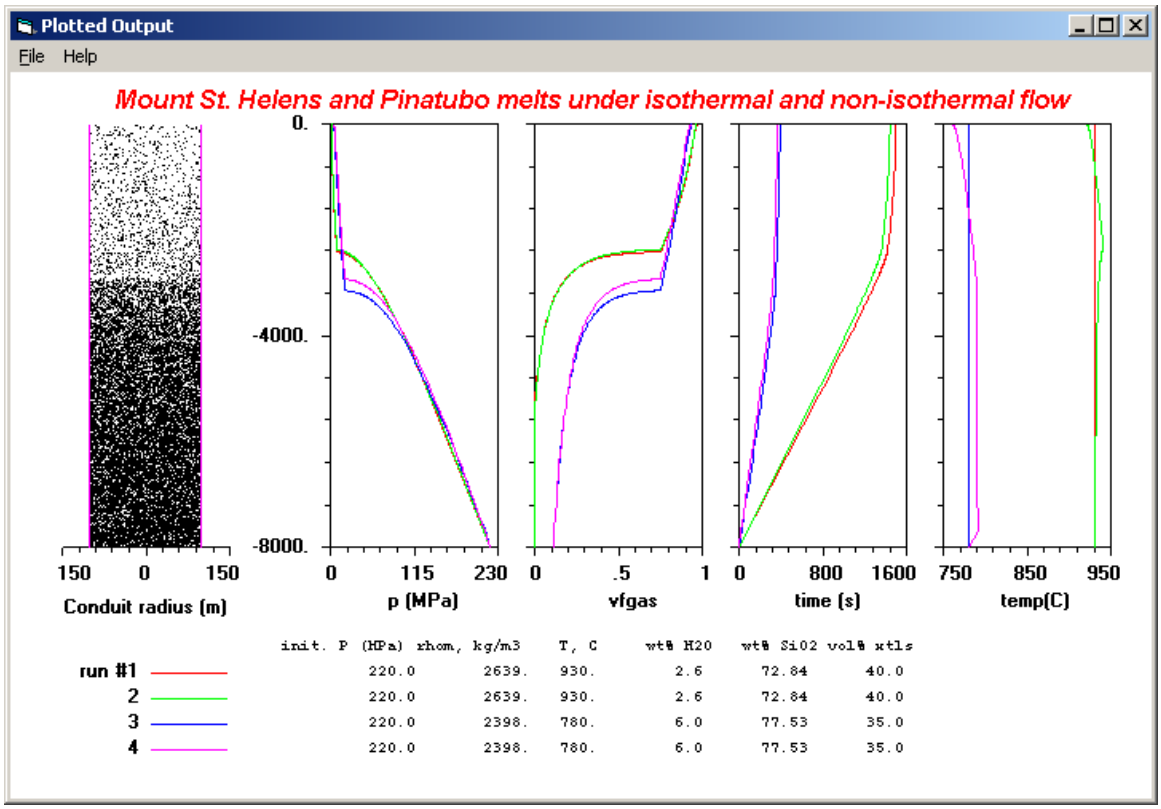


Figure 7. Plotted output showing the effect of adiabatic temperature change on conduit flow. Runs 1 and 2 use Mt. St. Helens magma (Table 2) with 40% plagioclase crystals, driven up an 8 km long, 53 m diameter conduit with a basal pressure of 220 MPa. These conditions give a mass eruption rate (2×10^7 kg s^{-1}) roughly equal to that of other estimates for the 18 May 1980 Mt. St. Helens Plinian eruption [*Carey and Sigurdsson*, 1985]. Run 1 assumes no temperature change; run 2 calculates temperature changes using the energy equation (equation (4)). Runs 3 and 4 use a Pinatubo magma containing 35% plagioclase crystals, driven up an 8 km long, 200 m diameter conduit with a basal pressure of 220 MPa [*Rutherford and Devine.*, 1996]. These input conditions give a mass eruption rate (8×10^8 kg s^{-1}) consistent with estimates based on pyroclast dispersal ($4-20 \times 10^8$ kg s^{-1} [*Koyaguchi*, 1996]). Run 3 assumes no temperature change; run 4 calculates temperature change using equation (4). At left in this figure is a pictographic representation of the volume-fraction gas in the mixture from the base to the top of the conduit, for run 4. Other plots are, from left to right, pressure (MPa), time (s) since entering the conduit, log viscosity (Pa s), and mixture temperature (C).

tions if conduit geometry is adjusted in a manner that minimizes flow resistance (i.e., by allowing the conduit to flare at the surface and optimizing the location of the maximum constriction). The degree to which geometry adjusts to optimize flow depends on the strength and mechanical properties of the host rock as well as intrusion history.

5.3. Effects of Adiabatic Temperature Change on Conduit Flow

[43] Most eruptive conduit models neglect temperature changes on the belief that they are minimal [e.g., *Wilson et al.*, 1980; *Dobran*, 1992; *Papale et al.*, 1998]. The degree to which temperature changes actually affect flow properties has not been rigorously tested, though thermal changes have been calculated for limiting thermodynamic conditions [*Mastin and Ghiorso*, 2001]. Temperature changes in the conduit are of two types: (1) in the lower conduit, viscous shearing dissipates momentum and heats the mixture and (2) near the surface, gas expansion, and exsolution cool the mixture. Near-surface gas expansion may cool the mixture by more than 200°C [*Mastin and Ghiorso*, 2001]; but most cooling occurs after magma has fragmented and therefore has little effect on flow properties. Viscous shearing may heat the mixture by only degrees to tens of degrees on average, but

heating may significantly affect flow if the viscosity is highly sensitive to temperature or if heat generation is concentrated at conduit margins rather than being distributed through the mixture.

[44] Figure 7 illustrates the effect of adiabatic temperature changes on two simulated eruptions: Mt. St. Helens on 18 May 1980 (runs 1 and 2) and Pinatubo on 15 June 1991 (runs 3 and 4). Runs 1 and 3 assume constant temperature; 2 and 4 calculate temperature changes as dictated by the energy equation (equation (4). For the case of Mt. St. Helens, the temperature increases gradually by $\sim 11°$ between the base of the conduit and the fragmentation depth; viscosity is $\sim$25% lower at the fragmentation depth than in the isothermal case; and mass eruption rate is $\sim$5% higher than in the isothermal case (2.10×10^7 versus 2.01×10^7 kg s^{-1}). For Pinatubo the effect is slightly greater: the temperature climbs rapidly by $\sim 10°$ within 250 m above the base of the conduit, then stabilizes. The viscosity at the fragmentation depth is $\sim$30% lower than in the isothermal case, and the mass eruption rate is $\sim$10% higher (9.8 versus 8.9×10^8 kg s^{-1}). The greater effect of viscous heating in the Pinatubo melt results from its lower temperature, higher viscosity, and the greater sensitivity of viscosity to temperature changes in this cooler melt. These effects are mitigated by lower crystallinity and larger conduit diameter, both of which reduce the overall friction factor.

[45] It should be noted that the crystallinity of both of these melts substantially exceeds that for which the Newtonian assumption and the *Marsh* [1981] calibration of the Roscoe-Einstein equation are accurate. In highly crystalline magmas, shear strain and associated heating may become concentrated within shear zones along the conduit margins. If shear heating at Pinatubo were limited to 10% of the conduit volume near its walls, the total heating in that shear zone would approach 100°, and its viscosity would decrease by an order of magnitude. Strain localization would likely produce pyroclasts that are texturally distinct from those in the less sheared center of the conduit. Whether non-Newtonian shearing contributed to the occurrence two texturally distinct pumice types at Pinatubo [*Pallister et al.*, 1996; *David et al.*, 1996] may be the subject of future study.

[46] For decades, viscous heating has been accepted as an important process in models of mafic dike flow [*Fujii and Uyeda*, 1974; *Carrigan et al.*, 1992; *Carrigan*, 2000]. One would expect viscous heating to be more important in silicic systems, given their higher viscosity. The effect of shear heating is probably greatest in shallow conduits beneath lava domes, where viscosity is extreme and crystallinity exceeds 60% [*Cashman*, 1992; *Melnik and Sparks*, 1999]. Modeling magma flow in these circumstances as isothermal and Newtonian may lead to gross errors.

6. Closing Comments

[47] Public programs will presumably increase in number and complexity in coming years with our ability to characterize and model complex processes. Although I will continually revise Conflow to improve its usability and accuracy, I hope that others will use and improve this code with time as well. Those who are interested in collaboration on improvements are encouraged to contact me.

Acknowledgments

[48] The documentation of this model and posting of its source code was initiated to satisfy the laudable requirement that all USGS Water Resources modeling studies use fully documented codes. After seeing my obscure USGS Open-File Report describing the model, Hubert Staudigel encouraged me to prepare this manuscript for G-cubed. The manuscript was significantly improved following reviews by Richard Iverson, Joseph Walder, Jon Major, Alex Proussevitch, and Don Dingwell. In addition, the code has been improved following suggestions from numerous users, notably Giovanni Macedonio, Ken Wohletz, Kathy Cashman, Margaret Mangan, and Michael Manga.

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